Reg. No. :

Question Paper Code : X 67617

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020

Fifth Semester

Computer Science and Engineering

MA 1301 – DISCRETE MATHEMATICS

(Regulations 2008)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. What is meant by proposition ? Give an example.

2. State the rules of inference for statement calculus.

3. Express the statement "Everybody loves somebody" using quantifiers.

4. Let D(u, υ) : u is divisible by υ. Over the universe {5, 7, 10, 11} what are the truth values of (∃ u) D(u, 5) and (y) D(y, 5).

5. Is the "divides" relation on the set of positive integers transitive ?

6. State any two properties of Lattices.

7. Define binary and n-ary operations.

8. Show that the function $f : R \to R$ defined by $f(x) = 2x + 7$ is a permutation function.

9. State the relation between semigroup and monoid.

10. What are "Encoder and Decoder" ?

PART – B **(5×16=80 Marks)**

11. a) i) Show that $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. **(8)**

ii) Obtain the principal conjunctive normal form of the formula given by $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$. **(8)**

(OR)

b) i) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$ and Q. **(8)**

ii) Using truth table, prove that $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$. **(8)**

12. a) i) Show, by that $(x)\,(P(x) \vee Q(x)) \Rightarrow (x)\,P(x) \vee (\exists x)\,Q(x)$. **(8)**

ii) Show that the premises "One student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high-paying job" imply the conclusion "Someone in this class can get a high-paying job". **(8)**

(OR)

b) i) Using CP or otherwise, obtain the following implication :
$(x)\,(P(x) \rightarrow Q(x)), (x)\,(R(x) \rightarrow \neg Q(x)) \Rightarrow (x)\,R(x) \rightarrow \neg P(x))$ **(8)**

ii) Show that $\neg P(a, b)$ follows logically from $(x)\,(y)\,(P(x, y) \rightarrow w(x, y))$ and $\neg w(a, b)$. **(8)**

13. a) i) Find the partitions of the set $A = \{1, 2, 3\}$. **(8)**

ii) In a distributive lattice L, prove that
$a \wedge b = a \wedge c$ and $a \vee b = a \vee c \Rightarrow b = c$. **(8)**

(OR)

b) i) Prove that the relation $R = \{(x, y)/x \equiv y \pmod 3\}$ defined on the set of real number is an equivalence relation. **(8)**

ii) In a Boolean algebra, prove that $(a+b)\cdot(a'+c) = ac + a'b$. **(8)**

14. a) i) Show that the functions f and g which both are from $N \times N$ to N given by $f(x, y) = x + y$ and $g(x, y) = xy$ are onto but not one-to-one. **(8)**

ii) Using characteristic function, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. **(8)**

(OR)

b) i) Show that the function [x/2] which is equal to the greatest integer which is $\leq x/2$ is primitive recursive. **(8)**

ii) Prove that the product of two even permutation is even. **(8)**

15. a) i) Define monoid. Give an example. For any commutative monoid (m, *), show that the set of idempotent elements of m forms a submonoid. **(8)**

ii) State and prove Lagrange's theorem. **(8)**

(OR)

b) i) Prove that the Kernel of a homomorphism g from a group (G, *) to (H, Δ) is a normal subgroup of G. **(8)**

ii) Determine the single-error correcting code generated by the parity-check matrix $H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$. If (0, 0, 0, 0, 1, 1) is the received word, find the transmitted code word. **(8)**

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